In the summer of 2024, I had the remarkable opportunity to participate in the Caltech Summer Undergraduate Research Fellowships (SURF) program. During this immersive experience, I undertook a research project in mathematical physics under the guidance of Professor Sergei Gukov. The focus of my research was on the effective central charge c_{eff} and resurgence analysis of BPS q-series associated with Brieskorn spheres, an area within the realm of theoretical physics.

The primary goal of my project was to investigate the effective central charge c_{eff} of a specific family of strongly coupled three-dimensional superconformal field theories, $T[-\Sigma(p_1, p_2, p_3)]$. These theories emerge from the 3d-3d correspondence, which connects certain three-dimensional field theories to three-manifolds. The Brieskorn spheres, represented as $\Sigma(p_1, p_2, p_3)$, are a class of three-manifolds that play a crucial role in this correspondence.

In particular, I examined the BPS q-series invariants, denoted as $\hat{Z}$, which are associated with these Brieskorn spheres. These invariants are expressed as linear combinations of false theta functions. Interestingly, for some orientation-reversal Brieskorn spheres $-\Sigma(p_1, p_2, p_3)$, the $\hat{Z}$ invariant is related to Ramanujan's mock theta functions. Both false theta functions and mock theta functions are types of q-series that are connected through a duality operation, $q \leftrightarrow 1/q$. Assuming that the coefficients of these q-series are integer-valued, I used numerical resurgence techniques to compute the dual false theta function, $\widetilde{\Psi}_p^{(a)}(q)^\vee$, for several pairs (p, a). This involved detailed and intricate calculations to understand the behavior and properties of these functions under the duality transformation.

Throughout the course of this project, I gained extensive hands-on experience in several advanced areas of mathematical physics. This included resurgence analysis, which involves studying the asymptotic behavior of series solutions to differential equations and their resurgence into exact solutions. Additionally, I delved into the study of three-manifolds, focusing on techniques such as surgery and plumbing, which are used to understand and construct different types of three-dimensional manifolds.

I also explored various knot invariants, such as the Alexander polynomial, the Jones polynomial, and the A-polynomial. These invariants are crucial in understanding the properties of three-manifolds constructed from surgery on knots and links.

Modularity was another area of focus, encompassing concepts like mock-modularity, quantum modularity, and the Eichler integral. These concepts are essential for understanding the mathematical structure of modular forms and their applications in theoretical physics. The WRT (Witten-Reshetikhin-Turaev) invariant and the $\hat{Z}$ invariant were also central to my research. The WRT invariant is a topological invariant of three-manifolds that has applications in quantum field theory and knot theory. The $\hat{Z}$ invariant, on the other hand, provides information about the invariants associated with Brieskorn spheres and their relation to false and mock theta functions.

This was my first time stepping into the field of mathematical physics. It provided me with a solid foundation in using Mathematica for complex calculations and simulations. Furthermore, it significantly broadened my understanding of theoretical physics and deepened my interest in resurgence analysis, bootstrapping methods, and strongly-coupled systems.

In addition to my academic pursuits, I took full advantage of my ten-week stay in Los Angeles to explore the city and its surroundings. One of the highlights was visiting Santa Monica Beach, where I enjoyed the stunning ocean views. As a film enthusiast, I was thrilled to explore Hollywood, the heart of the American film industry. I visited an academic museum that featured various filming props from iconic movies such as "Star Wars" and exhibitions on renowned directors Agnès Varda and Pedro Almodóvar, both of whom are among my favorites. The museum also provided insights into the history of the Oscars and the development of Hollywood as a major center for commercial cinema.

Another memorable experience was visiting Universal Studios Hollywood. The studio tour was a particular highlight, offering a behind-the-scenes look at movie production, including impressive CGI demonstrations and real sets used in popular movies and TV series. For instance, I had the chance to visit the set of the TV show "The Good Place" and the house from Alfred Hitchcock's classic film "Psycho." The experience of seeing the film industry up close was truly captivating.

During one of the weekends, my friends and I rented a car and took a trip to Joshua Tree National Park. Despite the scorching heat of 44 degrees Celsius, the desert landscape and expansive plains were breathtaking and offered a stark contrast to what I am accustomed to in Taiwan. That evening, we drove up to Mulholland Drive to take

in the stunning night views of Los Angeles.

Pasadena is a charming city with a variety of dining options. I enjoyed several great Asian restaurants, including Daisy Mint and Pho Banh Mi Che Cali, which helped alleviate my homesickness. One of my favorite places in Pasadena was the Norton Simon Museum. Admission is free, and the museum houses an impressive collection of paintings by artists such as Picasso, Seurat, and van Gogh. I also loved visiting the nearby Huntington Library, which features diverse gardens including a Chinese garden, a Japanese garden, and an Australian garden. It was a wonderful place to relax and unwind.

My time at Caltech was an incredibly rewarding experience. I lived in Page House with my roommate Kevin, who was from Harvard and interning at NASA Jet Propulsion Laboratory. I was fortunate to be assigned an office in the Downs and Lauritsen, where I worked alongside the high energy theory group. The environment was conducive to productivity, with ample blackboard space, coffee, and snacks available.

My daily routine involved starting the day with a visit to the gym, followed by

lunch at Browne. Their pizza was particularly delightful. I would then return to my office and work for several hours. For dinner, I frequented Red Door, where I enjoyed affordable and tasty options such as the grilled chicken sandwich and salmon salad. After dinner, I would often play table tennis with friends before heading back to my dorm for a shower and some rest.

Caltech's compact yet dynamic campus, close interdepartmental connections, and research-focused atmosphere made it an ideal environment for academic and personal growth. I feel incredibly grateful for the opportunity to conduct summer research at such a prestigious institution.

I would like to express my heartfelt thanks to Professor Miranda C. N. Cheng for connecting me with Professor Gukov, and to Professor Nathaniel Craig and Professor Pei-Ming Ho for their support through recommendation letters. I am also deeply appreciative of the invaluable mentorship by Professor Sergei Gukov and Mrunmay Jagadale. Finally, I am grateful to Dr. James Yen-Tang Oyang for his support throughout the program and to NTU OIA for their kind assistance. This summer has been a dream-like experience that has profoundly impacted my academic and personal journey.